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1. (a) Express $\frac{2}{4r^2 - 1}$ in partial fractions. (2)

(b) Hence use the method of differences to show that

$$\sum_{r=1}^n \frac{1}{4r^2 - 1} = \frac{n}{2n+1} \quad (3)$$



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2. Using algebra, find the set of values of x for which

$$3x - 5 < \frac{2}{x}$$

(5)



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3. (a) Find the general solution of the differential equation

$$\frac{dy}{dx} + 2y \tan x = e^{4x} \cos^2 x, \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

giving your answer in the form $y = f(x)$.

(6)

- (b) Find the particular solution for which $y = 1$ at $x = 0$

(2)



Question 3 continued

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4.

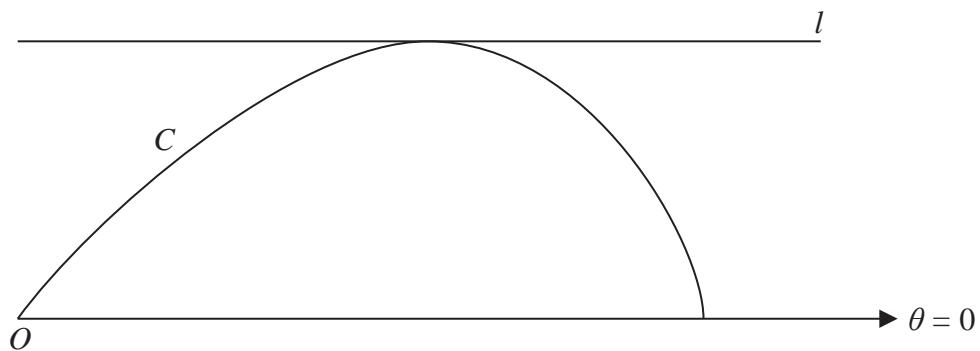


Figure 1

Figure 1 shows the curve C with polar equation

$$r = 2 \cos 2\theta, \quad 0 \leq \theta \leq \frac{\pi}{4}$$

The line l is parallel to the initial line and is a tangent to C .

Find an equation of l , giving your answer in the form $r = f(\theta)$.

(9)



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5. $y \frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 + 2y = 0$

- (a) Find an expression for $\frac{d^3y}{dx^3}$ in terms of $\frac{d^2y}{dx^2}$, $\frac{dy}{dx}$ and y .

(4)

Given that $y = 2$ and $\frac{dy}{dx} = 0.5$ at $x = 0$,

- (b) find a series solution for y in ascending powers of x , up to and including the term in x^3 .

(5)



Question 5 continued

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6. The transformation T maps points from the z -plane, where $z = x + iy$, to the w -plane, where $w = u + iv$.

The transformation T is given by

$$w = \frac{z}{iz + 1}, \quad z \neq i$$

The transformation T maps the line l in the z -plane onto the line with equation $v = -1$ in the w -plane.

- (a) Find a cartesian equation of l in terms of x and y .

(5)

The transformation T maps the line with equation $y = \frac{1}{2}$ in the z -plane onto the curve C in the w -plane.

- (b) (i) Show that C is a circle with centre the origin.

- (ii) Write down a cartesian equation of C in terms of u and v .

(6)



Question 6 continued

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7. (a) Use de Moivre's theorem to show that

$$\sin 5\theta \equiv 16 \sin^5 \theta - 20 \sin^3 \theta + 5 \sin \theta$$

- (b) Hence find the five distinct solutions of the equation

$$16x^5 - 20x^3 + 5x + \frac{1}{2} = 0$$

giving your answers to 3 decimal places where necessary.

(5)

- (c) Use the identity given in (a) to find

$$\int_0^{\frac{\pi}{4}} (4\sin^5 \theta - 5\sin^3 \theta) d\theta$$

expressing your answer in the form $a\sqrt{2} + b$, where a and b are rational numbers.

(4)



Question 7 continued

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8. (a) Show that the substitution $x = e^z$ transforms the differential equation

$$x^2 \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} - 2y = 3 \ln x, \quad x > 0 \quad (\text{I})$$

into the equation

$$\frac{d^2y}{dz^2} + \frac{dy}{dz} - 2y = 3z \quad (II)$$

- (b) Find the general solution of the differential equation (II).

(6)

- (c) Hence obtain the general solution of the differential equation (I) giving your answer in the form $y = f(x)$.

(1)



Question 8 continued

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Q8

(Total 14 marks)

TOTAL FOR PAPER: 75 MARKS

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